Chapter 8

Public Goods

Consider a company offering a fireworks display. Pretty much anyone nearby can watch the fireworks, and people with houses in the right place have a great view of them. The company that creates the fireworks can't compel those with nearby homes to pay for the fireworks, and so a lot of people get to watch them without paying. This will make it difficult or impossible for the fireworks company to make a profit.

8.1 Free Riders

LEARNING OBJECTIVE

1. What are people who just use public goods without paying called, and what is their effect on economic performance?

A **public good**[1] is a good that has two attributes: **Nonexcludability**[2], which means the producer can't prevent the use of the good by others, and **nonrivalry**[3], which means that many people can use the good simultaneously. The classic example of a public good is national defense. National defense is clearly nonexcludable because, if we spend the resources necessary to defend our national borders, it isn't going to be possible to defend everything except one apartment on the second floor of a three-story apartment building on East Maple Street. Once we have kept our enemies out of our borders, we've protected everyone within the borders. Similarly, the defense of the national borders exhibits a fair degree of nonrivalry, especially insofar as the strategy of defense is to deter an attack in the first place. That is, the same expenditure of resources protects all. It is theoretically possible to exclude some people from the use of a poem or a mathematical theorem, but exclusion is generally quite difficult. Both poems and theorems are nonrivalrous. Similarly, technological and software inventions are nonrivalrous, even though a patent grants the right to exclude the use by others. Another good that permits exclusion at a cost is a highway. A toll highway shows that exclusion is possible on the highways. Exclusion is quite expensive, partly because the tollbooths require staffing, but mainly because of the delays imposed on drivers associated with paying the tolls—the time costs of toll roads are high. Highways are an intermediate case where exclusion is possible only at a significant cost, and thus should be avoided if possible. Highways are also rivalrous at high-congestion levels, but nonrivalrous at low-congestion levels. That is, the marginal cost of an additional user is essentially zero for a sizeable number of users, but then marginal cost grows rapidly in the number of users. With fewer than 700 cars per lane per hour on a four-lane highway, generally the flow of traffic is unimpeded.The effect of doubling the number of lanes from two to four is dramatic. A two-lane highway generally flows at 60 mph or more provided there are fewer than 200 cars per lane per hour, while a four-lane highway can accommodate 700 cars per lane per hour at the same speed. As congestion grows beyond this level, traffic slows down and congestion sets in. Thus, west Texas interstate highways are usually nonrivalrous, while Los Angeles's freeways are usually very rivalrous.

1. A good that has the attributes of nonexclusivity and nonrivalry.
2. Condition in which the producer can't prevent the use of the good by others.
3. Condition in which many people can use a good simultaneously.

Like highways, recreational parks are nonrivalrous at low-use levels, becoming rivalrous as they become sufficiently crowded. Also like highways, it is possible, but expensive, to exclude potential users, since exclusion requires fences and a means for admitting some but not others. (Some exclusive parks provide keys to legitimate users, while others use gatekeepers to charge admission.)

Take the example of a neighborhood association that is considering buying land and building a park in the neighborhood. The value of the park is going to depend on the size of the park, and we suppose for simplicity that the value in dollars of the park to each household in the neighborhood is $S^b n^{-a}$, where n is the number of park users, S is the size of the park, and a and b are parameters satisfying $0 < a \le b < 1$. This functional form builds in the property that larger parks provide more value at a diminishing rate, but there is an effect from congestion. The functional form gives a reason for parks to be public—it is more efficient for a group of people to share a large park than for each individual to possess a small park, at least if $b > a$, because the gains from a large park exceed the congestion effects. That is, there is a scale advantage—doubling the number of people and the size of the park increases each individual's enjoyment.

How much will selfish individuals voluntarily contribute to the building of the park? That of course depends on what they think others will contribute. Consider a single household, and suppose that each household, i, thinks the others will contribute S_{-1} to the building of the park. Given this expectation, how much should each household, i, contribute? If the household contributes s, the park will have size $S = S_{-1} + s$, which the household values at $(S_{-1} + s)^b n^{-a}$. Thus, the net gain to a household that contributes s when the others contribute S_{-1} is $(S_{-1} + s)^b n^{-a} - s$.

Exercise 1 shows that individual residents gain from their marginal contribution if and only if the park is smaller than $S_0 = (bn^{-a})^{\frac{1}{1-b}}$. Consequently, under voluntary contributions, the only equilibrium park size is S_0. That is, for any park size smaller than S_0, citizens will voluntarily contribute to make the park larger. For any larger size, no one is willing to contribute.

Under voluntary contributions, as the neighborhood grows in number, the size of the park shrinks. This makes sense—the benefits of individual contributions to the park mostly accrue to others, which reduces the payoff to any one contributor.

How large should the park be? The total value of the park of size S to the residents together is n times the individual value, which gives a collective value of $S^b n^{1-a}$; and the park costs S, so from a social perspective the park should be sized to maximize $S^b n^{1-a} - S$, which yields an optimal park of size $S^* = (bn^{1-a})^{\frac{1}{1-b}}$. Thus, as the neighborhood grows, the park should grow, but as we saw, the park would shrink if the neighborhood has to rely on voluntary contributions. This is because people contribute individually, as if they were building the park for themselves, and don't account for the value they provide to their neighbors when they contribute. Under individual contributions, the hope that others contribute leads individuals not to contribute. Moreover, use of the park by others reduces the value of the park to each individual, so that the size of the park shrinks as the population grows under individual contributions. In contrast, the park ought to grow faster than the number of residents grows, as the per capita park size is $S^*/n = b^{\frac{1}{1-b}} n^{\frac{b-a}{1-b}}$,which is an increasing function of n.Reminder: In making statements like should and ought, there is no conflict in this model because every household agrees about the optimal size of the park, so that a change to a park size of S^*, paid with equal contributions, maximizes every household's utility.

The lack of incentive for individuals to contribute to a social good is known as a **free-rider problem**[4]. The term refers to the individuals who don't contribute to the provision of a public good, who are said to be **free riders**[5], that is, they ride freely on the contributions of others. There are two aspects of the free-rider problem apparent in this simple mathematical model. First, the individual incentive to contribute to a public good is reduced by the contributions of others, and thus individual contributions tend to be smaller when the group is larger. Put another way, the size of the free-rider problem grows as the community grows larger. Second, as the community grows larger, the optimal size of the public good grows. The market failure under voluntary contributions is greater as the community is larger. In the theory presented, the optimal size of the public good is $S^* = (bn^{1-a})^{\frac{1}{1-b}}$, and the actual size under voluntary contributions is $S^* = (bn^{1-a})^{\frac{1}{1-b}}$, a gap that gets very large as the number of people grows.

The upshot is that people will voluntarily contribute too little from a social perspective, by free riding on the contributions of others. A good example of the provision of public goods is a coauthored term paper. This is a public good because the grade given to the paper is the same for each author, and the quality of the paper depends on the sum of the efforts of the individual authors. Generally, with two authors, both work pretty hard on the manuscript in order to get a good grade. Add a third author, and it is a virtual certainty that two of the authors will think the third didn't work as hard and is a free rider on the project.

The term paper example also points to the limitations of the theory. Many people are not as selfish as the theory assumes and will contribute more than would be privately optimal. Moreover, with small numbers, bargaining between the

4. The lack of incentive for individuals to contribute to a social good.

5. Individuals who don't contribute to the provision of a public good.

contributors and the division of labor (each works on a section) may help to reduce the free-rider problem. Nevertheless, even with these limitations, the free-rider problem is very real; and it gets worse the more people are involved. The theory shows that if some individuals contribute more than their share in an altruistic way, the more selfish individuals contribute even less, undoing some of the good done by the altruists.

KEY TAKEAWAYS

- A public good has two attributes: nonexcludability, which means the producer can't prevent the use of the good by others; and nonrivalry, which means that many people can use the good simultaneously.
- Examples of public goods include national defense, fireworks displays, and mathematical theorems.
- Nonexcludability implies that people don't have to pay for the good; nonrivalry means that the efficient price is zero.
- A free rider is someone who doesn't pay for a public good.
- Generally voluntary contributions lead to too little provision of public goods.
- In spite of some altruism, the free-rider problem is very real, and it gets worse the more people are involved.

EXERCISES

1. Verify that individual residents gain from contributing to the park if $S < (bn^{-a})^{\frac{1}{1-b}}$ and gain from reducing their contributions if $S > (bn^{-a})^{\frac{1}{1-b}}$.
2. For the model presented in this section, compute the elasticity of the optimal park size with respect to the number of residents—that is, the percentage change in S^* for a small percentage change in n. [Hint: Use the linear approximation trick $(1+\Delta)^r \approx r\Delta$ for Δ near zero.]
3. For the model presented in this section, show that an individual's utility when the park is optimally sized and the expenses are shared equally among the n individuals is

$$u = \left(b^{\frac{b}{1-b}} - b^{\frac{1}{1-b}}\right) n^{\frac{b-a}{1-b}}.$$

   Does this model predict an increase in utility from larger communities?
4. Suppose two people, Person 1 and Person 2, want to produce a playground to share between them. The value of the playground of size S to each person is $\sqrt{S}$, where S is the number of dollars spent to build it. Show that, under voluntary contributions, the size of the playground is ¼ and that the efficient size is 1.
5. For the previous exercise, now suppose Person 1 offers "matching funds"—that is, offers to contribute an equal amount to the contributions of Person 2. How large a playground will Person 2 choose?

8.2 Provision With Taxation

LEARNING OBJECTIVE

1. If people won't pay for public goods, can society tax them instead?

Faced with the fact that voluntary contributions produce an inadequate park, the neighborhood turns to taxes. Many neighborhood associations or condominium associations have taxing authority and can compel individuals to contribute. One solution is to require each resident to contribute the amount 1, resulting in a park that is optimally sized at *n*, as clearly shown in the example from the previous section. Generally it is possible to provide the correct size of the public good using taxes to fund it. However, this is challenging in practice, as we illustrate in this slight modification of the previous example.

Let individuals have different strengths of preferences, so that individual *i* values the public good of size *S* at an amount $v_i S^b n^{-a}$ that is expressed in dollars. (It is useful to assume that all people have different *v* values to simplify arguments.) The optimal size of the park for the neighborhood is $n^{\frac{-a}{1-b}}\left(b\sum_{i=1}^{n} v_i\right)^{\frac{1}{1-b}} = (b\overline{v})^{\frac{1}{1-b}} n^{\frac{1-a}{1-b}}$, where $\overline{v} = \frac{1}{n}\sum_{i=1}^{n} v_i$is the average value. Again, taxes can be assessed to pay for an optimally sized park, but some people (those with small *v* values) will view that as a bad deal, while others (with large *v*) will view it as a good deal. What will the neighborhood choose to do?

If there are an odd number of voters in the neighborhood, we predict that the park size will appeal most to the **median voter**[6].The voting model employed here is that there is a status quo, which is a planned size of *S*. Anyone can propose to change the size of *S*, and the neighborhood then votes yes or no. If an *S* exists such that no replacement gets a majority vote, that *S* is an equilibrium under majority voting. This is the voter whose preferences fall in the middle of the range. With equal taxes, an individual obtains $v_i S^b n^{-a} - S/n$ .If there are an odd number of people, *n* can be written as $2k + 1$. The median voter is the person for whom there are *k* values *vi* larger than hers and *k* values smaller than hers. Consider increasing *S*. If the median voter likes it, then so do all the people with higher *v*'s, and the proposition to increase *S* passes. Similarly, a proposal to decrease *S* will get a majority if the median voter likes it. If the median voter likes reducing *S*, all the individuals with smaller *vi* will vote for it as well. Thus, we can see the preferences of the median

6. The voter whose preferences fall in the middle of the range.

voter are maximized by the vote, and simple calculus shows that this entails $S = (bv_k)^{\frac{1}{1-b}} n^{\frac{1-a}{1-b}}$.

Unfortunately, voting does not result in an efficient outcome generally and only does so when the average value equals the median value. On the other hand, voting generally performs much better than voluntary contributions. The park size can either be larger or smaller under median voting than is efficient.The general principle here is that the median voting will do better when the distribution of values is such that the average of *n* values exceeds the median, which in turn exceeds the maximum divided by *n*. This is true for most empirically relevant distributions.

KEY TAKEAWAYS

- Taxation—forced contribution—is a solution to the free-rider problem.
- An optimal tax rate is the average marginal value of the public good.
- Voting leads to a tax rate equal to the median marginal value, and hence does not generally lead to efficiency, although it outperforms voluntary contributions.

EXERCISES

1. Show for the model of this section that, under voluntary contributions, only one person contributes, and that person is the person with the largest *vi*. How much do they contribute? [Hint: Which individual *i* is willing to contribute at the largest park size? Given the park that this individual desires, can anyone else benefit from contributing at all?]
2. Show that, if all individuals value the public good equally, voting on the size of the good results in the efficient provision of the public good.

8.3 Local Public Goods

LEARNING OBJECTIVE

1. What can we do if we disagree about the optimal level of public goods?

The example in the previous section showed the challenges to a neighborhood's provision of public goods created by differences in the preferences. Voting does not generally lead to the efficient provision of the public good and does so only rarely when all individuals have the same preferences.

A different solution was proposed by TieboutCharles Tiebout, 1919–1962. His surname is pronounced "tee-boo." in 1956, which works only when the public goods are local. People living nearby may or may not be excludable, but people living farther away can be excluded. Such goods that are produced and consumed in a limited geographical area are **local public goods**[7]. Schools are local—more distant people can readily be excluded. With parks it is more difficult to exclude people from using the good; nonetheless, they are still local public goods because few people will drive 30 miles to use a park.

Suppose that there are a variety of neighborhoods, some with high taxes, better schools, big parks, beautifully maintained trees on the streets, frequent garbage pickup, a first-rate fire department, extensive police protection, and spectacular fireworks displays, and others with lower taxes and more modest provision of public goods. People will move to the neighborhood that fits their preferences. As a result, neighborhoods will evolve with inhabitants that have similar preferences for public goods. Similarity among neighbors makes voting more efficient, in turn. Consequently, the ability of people to choose their neighborhoods to suit their preferences over taxes and public goods will make the neighborhood provision of public goods more efficient. The "Tiebout theory" shows that local public goods tend to be efficiently provided. In addition, even private goods such as garbage collection and schools can be efficiently publicly provided when they are local goods, and there are enough distinct localities to offer a broad range of services.

7. Goods that are produced and consumed in a limited geographical area.

KEY TAKEAWAYS

- When public goods are local—people living nearby may or may not be excludable, whereas people living farther away may be excluded—the goods are "local public goods."
- Specialization of neighborhoods providing in distinct levels of public goods, when combined with households selecting their preferred neighborhood, can lead to efficient provision of public goods.

EXERCISES

1. Consider a babysitting cooperative, where parents rotate supervision of the children of several families. Suppose that, if the sitting service is available with frequency Y, a person's i value is viY and the costs of contribution y is ½ ny^2, where y is the sum of the individual contributions and n is the number of families. Rank $v_1 \geq v_2 \geq \ldots \geq v_n$.

   a. What is the size of the service under voluntary contributions?

      (Hint: Let yi be the contribution of family i. Identify the payoff of family j as
      $v_j\left(y_j + \sum_{i \neq j} y_i\right) - ½n\left(y_j\right)^2$.

      What value of yj maximizes this expression?)

   b. What contributions maximize the total social value

   c. $\left(\sum_{j=1}^{n} v_j\right)\left(\sum_{j=1}^{n} y_j\right) - ½n \sum_{i=1}^{n} \left(y_j\right)^2$?

   d. $yi\ i$

   e. Let $\mu = \frac{1}{n}\sum_{j=1}^{n} v_j$ and $\sigma^2 = \frac{1}{n}\sum_{j=1}^{n}(v_j - \mu)^2$.

      Conclude that, under voluntary contributions, the total value generated by the cooperative is $\frac{n}{2}\left(\mu^2 - \sigma^2\right)$.

      (Hint: It helps to know that
      $\sigma^2 = \frac{1}{n}\sum_{j=1}^{n}(v_j - \mu)^2 = \frac{1}{n}\sum_{j=1}^{n} v_j^2 - \frac{2}{n}\sum_{j=1}^{n}\mu v_j + \frac{1}{n}\sum_{j=1}^{n}\mu^2 =$
)